College Algebra and Trigonometry

Ratti | McWaters | Skrzypek

Fourth Edition

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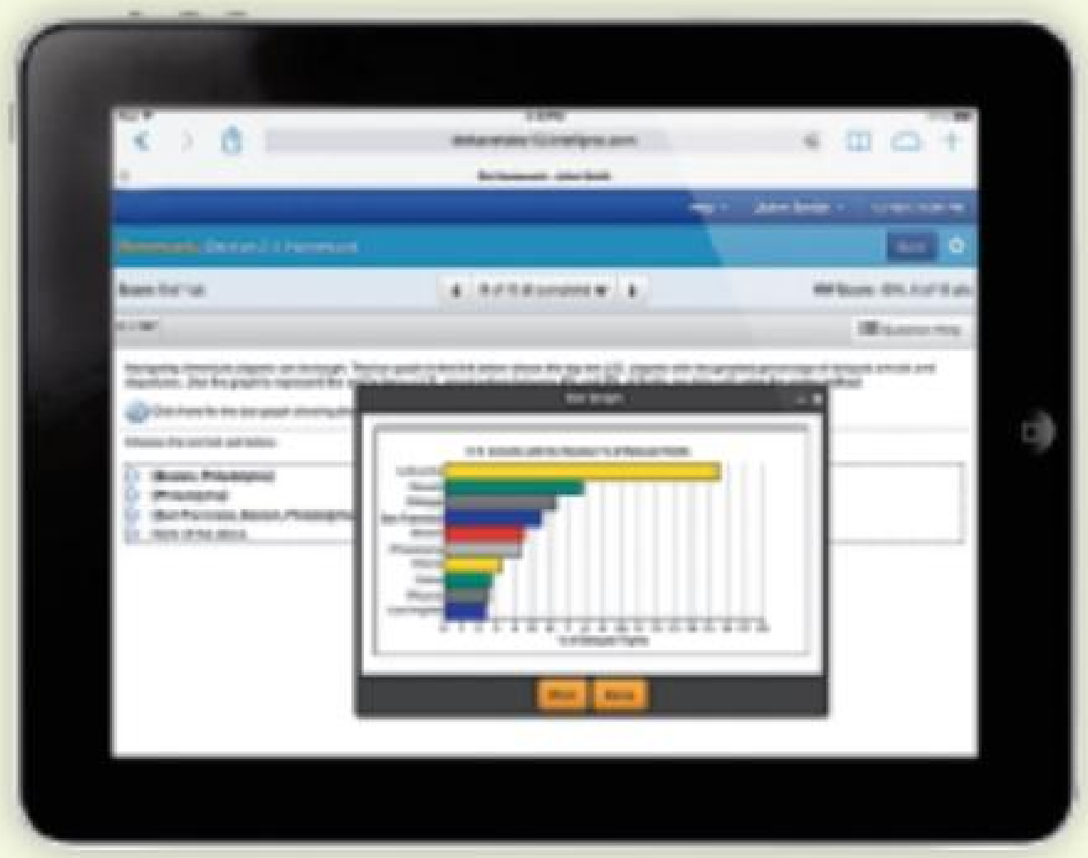
MyLab™ Math, Pearson's online homework, tutorial, and assessment program, creates personalized experiences for students and provides powerful tools for instructors. With a wealth of tested and proven resources, each course can be tailored to fit your specific needs.



Learning In Any Environment

- Because classroom formats and student needs continually change and evolve, MyLab Math has built-in flexibility to accommodate various course designs and formats.
- With an updated and streamlined design, students and instructors can access MyLab Math from most mobile devices.

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Not every student learns the same way or at the same rate. Thanks to our advances in adaptive learning technology capabilities, you no longer have to teach as if they do.

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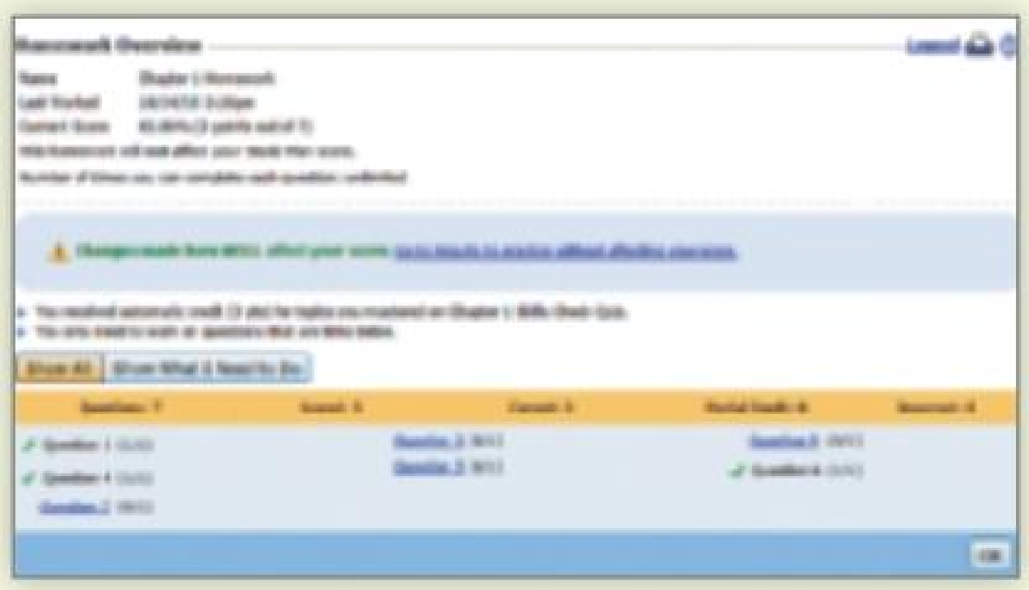
- MyLab Math can **personalize homework assignments** for students based on their performance on a test or quiz. This way, students can focus on just the topics they have not yet mastered.



- New - **Skill Builder** exercises offer just-in-time adaptive practice from within homework assignments to help students build the skills needed to successfully complete their work.

Visit pearson.com/mylab/math and click Get Trained to make sure you're getting the most out of MyLab Math.

MyLab Math Online Course for College Algebra and Trigonometry by Ratti, McWaters, Skrzypek (access code required)

Pearson MyLab

Achieve Your Potential

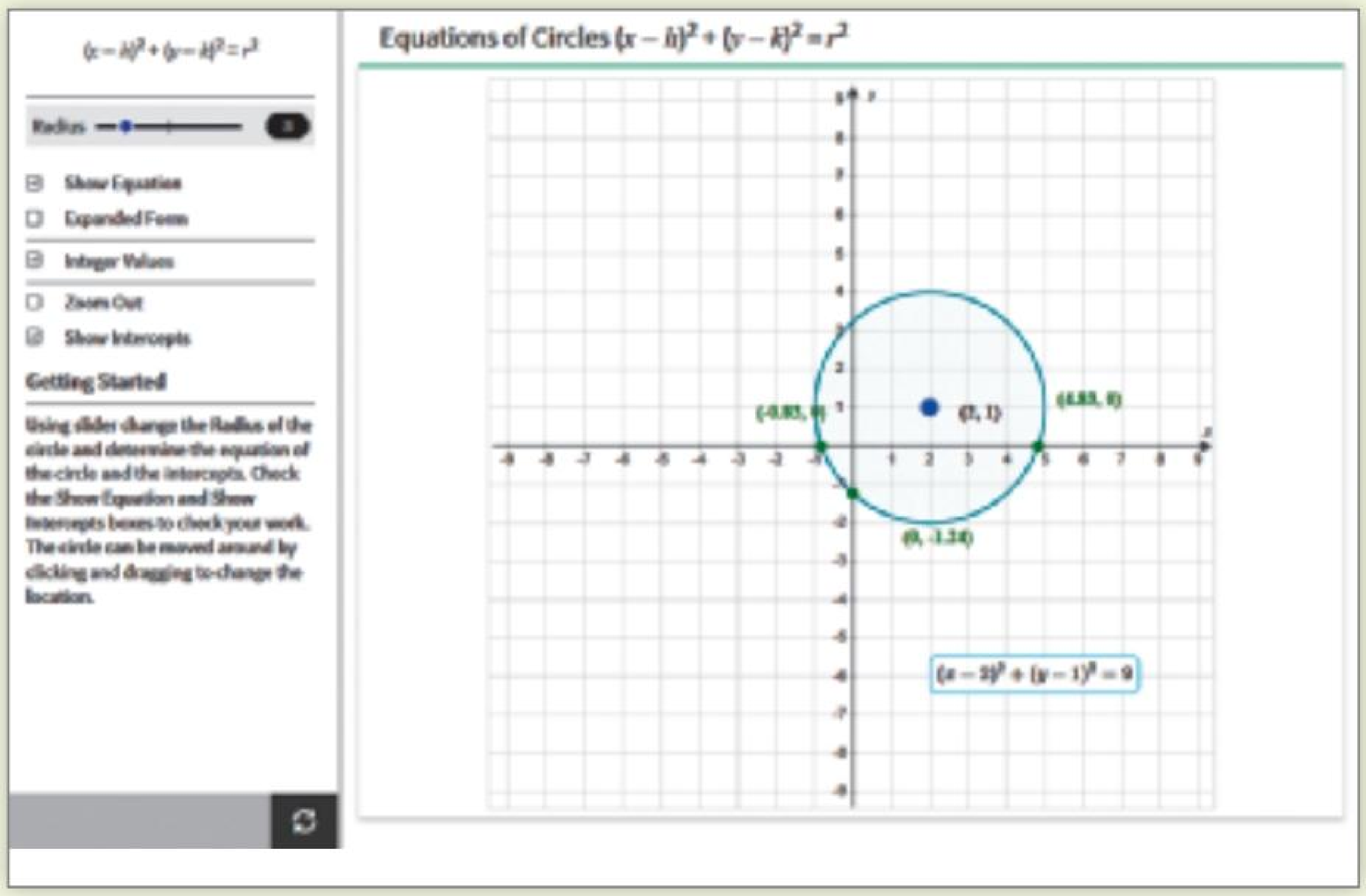
Success in math can make a difference in life. MyLab Math is a learning program with resources to help students achieve their potential in this course and beyond. MyLab Math helps you get up to speed on course material and understand how math will play a role in future careers.

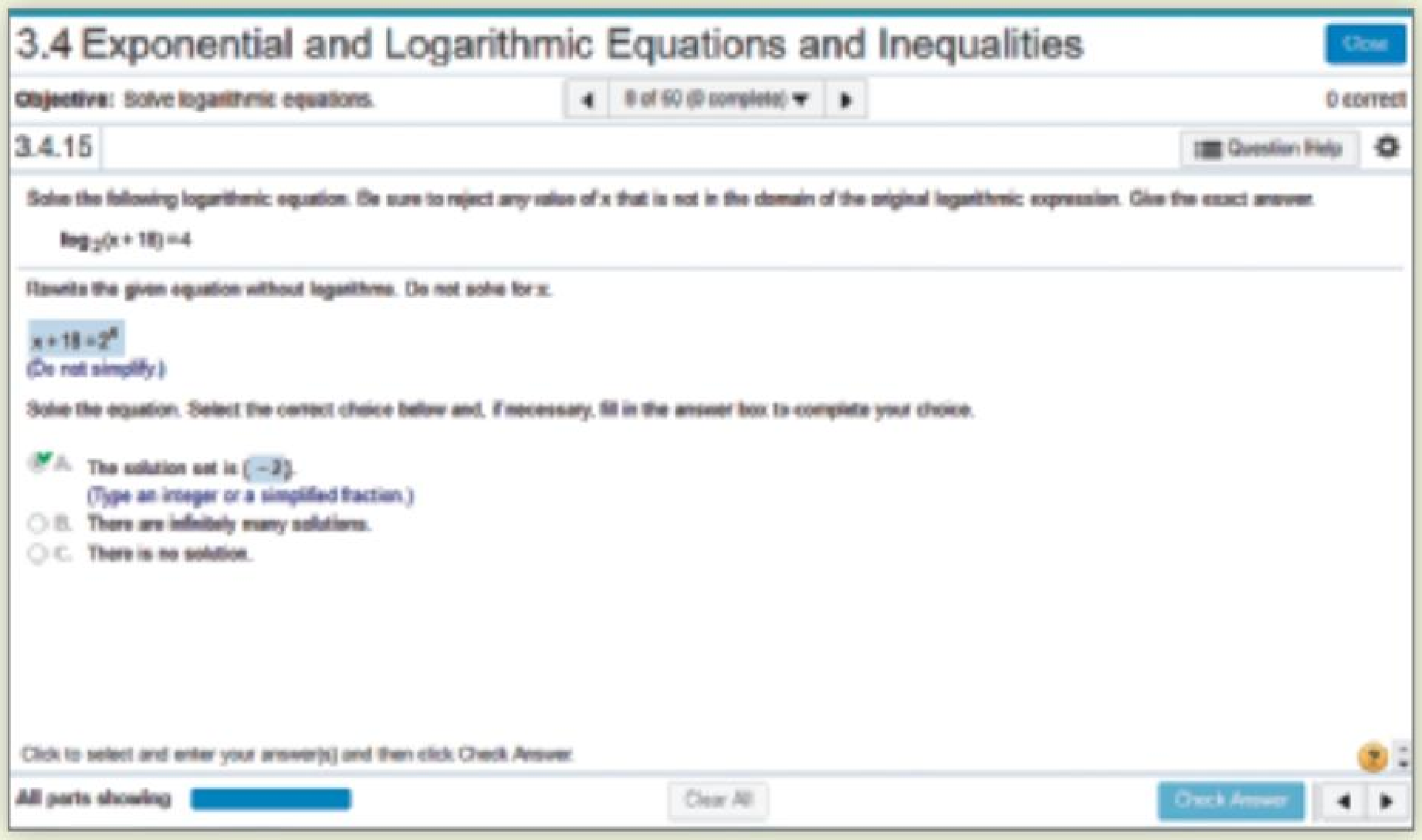
Visualization and Conceptual Understanding

These new MyLab Math resources aid in visual thinking, conceptual understanding and connecting mathematical concepts.

Guided Visualizations

Engaging interactive figures bring mathematical concepts to life, helping you visualize the concepts through directed explorations and purposeful manipulation. Guided Visualizations can be assigned to encourage active learning, critical thinking, and conceptual understanding.





Setup and Solve Exercises

Stepped-out exercises ask students to first describe how they will set up and approach the problem. This reinforces conceptual understanding of the process applied in solving the problem and promotes long term retention of the skill. Access to the eText is available for additional support.

pearson.com/mylab/math

Connect the Concepts and Relate the Math

Preparedness

One of the biggest challenges in Precalculus is being adequately prepared for the course with prerequisite knowledge. MyLab Math's learning resources help refresh knowledge of topics previously learned that are necessary to be successful in Precalculus. Brushing up on these essential algebra skills in each section can dramatically help increase success.

Due	Assignment
	Getting Ready for Chapter 1 Quiz
	Getting Ready for Chapter 1 Homework
	Getting Ready for Chapter 2 Quiz
	Getting Ready for Chapter 2 Homework
	Getting Ready for Chapter 3 Quiz
	Getting Ready for Chapter 3 Homework
	Getting Ready for Chapter 4 Quiz
	Getting Ready for Chapter 4 Homework

Getting Ready

Getting Ready material provides just-in-time review, integrated throughout the course as needed to prepare students with prerequisite material to succeed. From a quick quiz, a personalized, just-in-time review assignment is generated for each student, allowing them to refresh forgotten concepts.

Maintaining Skills

Our authors are committed to students—helping them retain essential information and maintain skills needed for success in current, as well as, future math courses.

Ongoing Review

Exercises written to help maintain skills and prepare for the next section encourage review of key mathematical concepts throughout the course. This ongoing review keeps knowledge fresh, prepares for new material learning, and promotes knowledge retention for the current course and for future courses.

Getting Ready for the Next Section

In Exercises 121–126, consider triangle ABC shown in the adjoining figure. Use the Pythagorean theorem to find the third side from the given two sides.

121. $a = 5, b = 12$

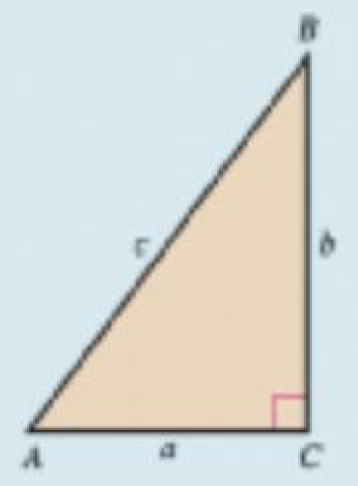
122. $a = 12, c = 20$

123. $b = 6, c = 10$

124. $a = 20, b = 21$

125. $b = 3, c = 6$

126. $a = 5, b = 10$



In Exercises 127 and 128, find $\frac{A}{B}$ from the given values of A and B. Then rationalize the denominator of the quotient.

127. $A = \frac{1}{2}, B = \frac{\sqrt{3}}{2}$

128. $A = \frac{\sqrt{3}}{2}, B = \sqrt{\frac{3}{2}}$

College Algebra and Trigonometry

Fourth Edition

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Cover Image: Primeimages/E+/Getty Images

Library of Congress Cataloging-in-Publication Data

Names: Ratti, J. S., author. | McWaters, Marcus, author. | Skrzypek, Leslaw, author.
Title: College algebra and trigonometry / J.S. Ratti, University of South Florida, Marcus McWaters, University of South Florida, Lesław A. Skrzypek, University of South Florida.
Description: Fourth edition. | Boston : Pearson Education, Inc., 2017. | Includes index.
Identifiers: LCCN 2017038506 | ISBN 9780134696478 (hardcover)
Subjects: LCSH: Algebra–Textbooks. | Trigonometry–Textbooks.
Classification: LCC QA152.3 .R38 2017 | DDC 512/.13–dc23 LC record available at https://lccn.loc.gov/2017038506

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ISBN 13: 978-0-13-469647-8
ISBN 10: 0-13-469647-6

FOREWORD

We're pleased to present the fourth edition of *College Algebra and Trigonometry*. Our experience in teaching this material at the University of South Florida has been exceptionally rewarding. Because students are accustomed to information being delivered by electronic media, the introduction of MyLab™ Math into our courses was, and remains, seamless. We hope you will have a similar experience.

Today's college algebra and trigonometry students and instructors face many challenges. Students arrive with various levels of comprehension from their previous courses. Instead of really learning the concepts presented, students often resort to memorization to pass the course. As a result, a course needs to establish a common starting point for students and engage them in becoming active learners, without sacrificing the solid mathematics essential for conceptual understanding. Instructors in this course must take on the task of providing students with an understanding of college algebra and trigonometry, preparing them for the next step, and ensuring that they find mathematics useful and interesting. Our efforts in this direction have been aided considerably by the many suggestions we have received from users of the first, second, and third editions of this text.

This text places a strong emphasis on both concept development and real-life applications. Topics such as functions, graphing, the difference quotient, and limiting processes provide thorough preparation for the study of calculus and will improve students' comprehension of algebra. Just-in-time review throughout the text ensures that all students are brought to the same level before being introduced to new concepts. Numerous applications motivate students to apply the concepts and skills they learn in college algebra and trigonometry to other courses, including the physical and biological sciences, engineering, and economics, and to on-the-job and everyday problem solving. Students are given ample opportunities in this course to think about important mathematical ideas and to practice and apply algebraic skills.

Throughout the text, we emphasize why the material being covered is important and how it can be applied. By thoroughly developing mathematical concepts with clearly defined terminology, students see the "why" behind those concepts, paving the way for a deeper understanding, better retention, less reliance on rote memorization, and ultimately more success. The level of exposition was selected so that the material is accessible to students and provides them with an opportunity to grow.

It is our hope that once you have read through our text, you will see that we were able to fulfill our initial goals of writing for today's students and for you, the instructor.

Marcus McWaters

Marcus McWaters

Lesław Skrzypek

JSRatti

J. S. Ratti

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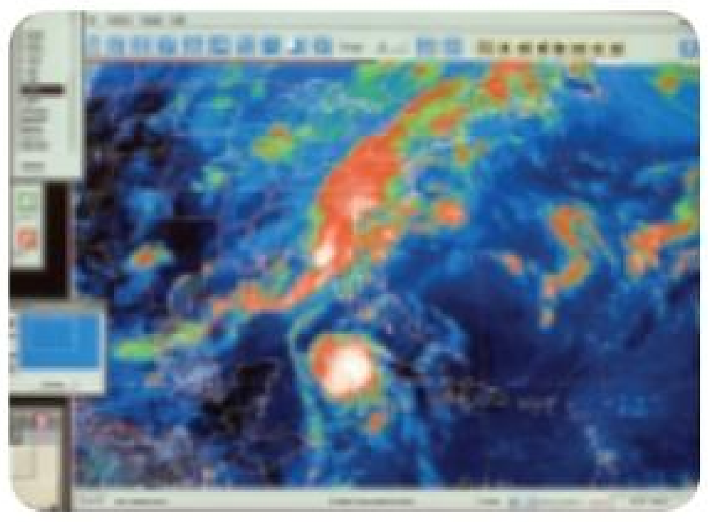
Chapter 1



Chapter 2

Chapter 3

Chapter 4

Chapter 5

Chapter 6

Chapter 7

Chapter 8

Chapter 9

Chapter 10

Chapter 11

PREFACE

Students begin college algebra and trigonometry classes with widely varying backgrounds. Some haven't taken a math course in several years and may need to spend time reviewing prerequisite topics, while others are ready to jump right into new and challenging material. In Chapter P and in some of the early sections of other chapters, we have provided review material in such a way that it can be used or omitted as appropriate for your course. In addition, students may follow several paths after completing a college algebra and trigonometry course. Many will continue their study of mathematics in courses such as finite mathematics, statistics, and calculus. For others, college algebra and trigonometry may be their last mathematics course.

Responding to the current and future needs of all of these students was essential in creating this text. We introduce each exercise set with several concept and vocabulary exercises, consisting of fill-in-the-blank and true-false exercises. They are not computation-reliant, but rather test whether students have absorbed the basic concepts and vocabulary of the section. Exercises asking students to extrapolate information from a given graph now appear in much greater number and depth throughout the course. We continue to present our content in a systematic way that illustrates how to study and what to review. We believe that if students use this material well, they will succeed in this course. The changes in this edition result from the thoughtful feedback we have received from students and instructors who have used previous editions of the text. This feedback crucially enhances our own experiences, and we are extremely grateful to the many contributors whose insights are reflected in this new edition.

Key Content Changes

EXERCISES We continue to improve the balance of exercises, providing a smother transition from the less challenging to the more challenging exercises.

Concept and Vocabulary Exercises. Each exercise section begins with exercises that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension of the material in that section.

Exercises Preparing Students for Material in the Next Section. Each exercise section ends with a set of exercises titled **Getting Ready for the Next Section** that provides a review of the concepts and skills that will be used in the following section.

Graph and Data-Related Exercises. We have introduced exercises throughout the text that demonstrate how to extract information about real-world situations from a graphical representation of that situation, as well as how to recover algebraic or trigonometric formulations of a graph by using key characteristics of that graph.

Modeling Exercises. A section on building linear, exponential, logarithmic, and power models from data was added in Chapter 4; this section contains new exercises using each type of model.

Overall, approximately 20% of the exercises have been updated, and more than 500 brand-new exercises have been added. These new exercises primarily consist of applications that connect with students' everyday experiences and enhance students' understanding of graphing.

CHAPTER 1

- Added a separate section, **Applications of Linear Equations: Modeling**, as Section 1.2. This moved some material from Section 1.1 on linear equations in the previous edition into Section 1.2, providing two relatively short sections that are more easily covered in one lecture.
- Introduced the discriminant, showing how to determine the number and type of solutions (rational or irrational) to a quadratic equation having integer coefficients.
- Provided simpler introductory examples for material students typically find difficult.

CHAPTER 2

- Introduced "delta x" and "delta y" notation when we discussed the slope of a line.
- Expanded the discussion on modeling data using linear regression.
- Added real-world examples to the discussion of increasing and decreasing functions and finding maxima and minima, and expanded the discussion on turning points.
- Added real-world examples to the discussion of average rate of change.

CHAPTER 3

- A **Summary of Main Facts** has been added to the section on quadratic functions.
- Expanded the discussion on functions of even and odd degree and on the end behavior of polynomial functions.
- Added an example showing how to graph a polynomial given in factored form.
- Expanded the discussion on horizontal asymptotes, along with improved graphics.

CHAPTER 4

- Added a schematic showing various transformations of the graph of $f(x) = e^x$.
- Expanded the discussion of exponential growth and decay.
- Added a schematic showing various transformations of the graph of $f(x) = \log_a x$.
- Added a comparison of the end behavior (as x approaches infinity) of the exponential, logarithmic and linear functions.
- Added a procedure for solving logarithmic equations using exponential form.
- Added material on building linear, exponential, logarithmic, and power models from data.

CHAPTER 5

- Provided an alternative way of graphing trigonometric functions by using transformations of functions and added a subsection on the even–odd properties of the trigonometric functions.

CHAPTER 6

- The sections on trigonometric equations were completely rewritten.

CHAPTER 7

- Added a procedure showing how to use the Law of Cosines to solve SSA triangles.
- Added a summary of the procedures for solving oblique triangles.
- Added a subsection showing how to use area formulas to find the altitude of a triangle.

CHAPTER 8

- Added a summary of the methods for solving three equations in three unknowns.
- Added an example showing how to find a partial-fraction decomposition when the denominator has repeated linear factors.

CHAPTER 9

- Added a schematic showing the most common transformations and their corresponding matrices.
- Added a discussion about the connection between combining transformations and multiplying their corresponding matrices and showing that the order in which the transformations are performed matters.

CHAPTER 10

- Sections 10.2, 10.3, and 10.4 have additional examples showing how to obtain the equation of the conic discussed in that section from key characteristics of its graph.

CHAPTER 11

- Expanded discussion, with examples, showing the connection between arithmetic sequences and linear functions and between geometric sequences and exponential functions.

Features

CHAPTER OPENER Each chapter opener includes a description of applications (one of them illustrated) relevant to the content of the chapter and the list of topics that will be covered. In one page, students see what they are going to learn and why they are learning it.

SECTION OPENER WITH APPLICATION Each section opens with a list of prerequisite topics, complete with section and page references, which students can review prior to starting the section. The **Objectives** of the section are also clearly stated and numbered, and then referenced again in the margin of the lesson at the point where the objective's topic is taught. An **Application** containing a motivating anecdote or an interesting problem then follows. An example later in the section relating to this application and identified by the same icon ( ) is then solved using the mathematics covered in the section. These applications utilize material from a variety of fields: the physical and biological sciences (including health sciences), economics, art and architecture, history, and more.

EXAMPLES AND PRACTICE PROBLEMS **Examples** include a wide range of computational, conceptual, and modern applied problems carefully selected to build confidence, competency, and understanding. Every example has a title indicating its purpose and presents a detailed solution containing annotated steps. All examples are followed by a **Practice Problem** for students to try so that they can check their understanding of the concept covered. Answers to the Practice Problems are provided just before the section exercises.

PROCEDURE BOXES These boxes, interspersed throughout the text, present important procedures in numbered steps. Special **Procedure in Action** boxes present important multistep procedures, such as the steps for doing synthetic division, in a two-column format. The steps of the procedure are given in the left column, and an example is worked, following these steps, in the right column. This approach provides students with a clear model with which they can compare when encountering difficulty in their work. These boxes are a part of the numbered examples.

ADDITIONAL PEDAGOGICAL FEATURES

Definitions, ***Theorems***, ***Properties***, and ***Rules*** are all boxed and titled for emphasis and ease of reference.

Warnings appear as appropriate throughout the text to apprise students of common errors and pitfalls that can trip them up in their thinking or calculations.

Summary of Main Facts boxes summarize information related to equations and their graphs, such as those of the conic sections.

A Calculus Symbol appears next to information in the text that is essential for the study of calculus.

MARGIN NOTES

Side Notes provide hints for handling newly introduced concepts.

Recall notes remind students of a key idea learned earlier in the text that will help them work through a current problem.

Technology Connections give students tips on using calculators to solve problems, check answers, and reinforce concepts. Note that the use of graphing calculators is optional in this text.

Do You Know? features provide students with additional interesting information on topics to keep them engaged in the mathematics presented.

Historical Notes give students information on key people or ideas in the history and development of mathematics.

EXERCISES The heart of any textbook is its exercises, so we have tried to ensure that the quantity, quality, and variety of exercises meet the needs of all students. Exercises are carefully graded to strengthen the skills developed in the section and are organized using the following categories. **Concepts and Vocabulary** exercises begin each exercise set with problems that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension of the material in that section. **Building Skills** exercises develop fundamental skills—each odd-numbered exercise is closely paired with its consecutive even-numbered exercise. **Applying the Concepts** features use the section's material to solve real-world problems—all are titled and relevant to the topics of the section. **Beyond the Basics** exercises provide more challenging problems that give students an opportunity to reach beyond the material covered in the section— these are generally more theoretical in nature and are suitable for honors students, special assignments, or extra credit. **Critical Thinking/Discussion/Writing** exercises, appearing as appropriate, are designed to develop students' higher-level thinking skills. Calculator problems, identified by , are included where needed. **Getting Ready for the Next Section** exercises end each exercise set with problems that provide a review of concepts and skills that will be used in the following section.

CHAPTER REVIEW AND TESTS The chapter-ending material begins with an extensive **Review** featuring a two-column, section-by-section summary of the definitions, concepts, and formulas covered in that chapter, with corresponding examples. This review provides a description and examples of key topics indicating where the material occurs in the text, and encourages students to reread sections rather than memorize definitions out of context. **Review Exercises** provide students with an opportunity to practice what they have learned in the chapter. Then students are given two chapter test options. They can take **Practice Test A** in the usual open-ended format and/or **Practice Test B**, covering the same topics, in a multiple-choice format. All tests are designed to increase student comprehension and verify that students have mastered the skills and concepts in the chapter. Mastery of these materials should indicate a true comprehension of the chapter and the likelihood of success on the associated in-class examination. **Cumulative Review Exercises** appear at the end of every chapter, starting with Chapter 2, to remind students that mathematics is not modular and that what is learned in the first part of the book will be useful in later parts of the book and on the final examination.

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Get the Most Out of MyLab Math

Used by over 3 million students a year, MyLab™ Math is the world's leading online program for teaching and learning mathematics. MyLab Math delivers assessment, tutorials, and multimedia resources that provide engaging and personalized experiences for each student, so learning can happen in any environment. Each course is developed to accompany Pearson's best-selling content, authored by thought leaders across the math curriculum, and can be easily customized to fit any course format.

Preparedness

One of the biggest challenges in many mathematics courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math offers a variety of content and course options to support students with just-in-time remediation and key-concept review.

- **Getting Ready** material provides just-in-time review, integrated throughout the course as needed to prepare students with prerequisite material to succeed. From a quick quiz, a personalized, just-in-time review assignment is generated for each student, allowing them to refresh forgotten concepts.

Due	Assignment
	Q Getting Ready for Chapter 1 Quiz
	H Getting Ready for Chapter 1 Homework
	Q Getting Ready for Chapter 2 Quiz
	H Getting Ready for Chapter 2 Homework
	Q Getting Ready for Chapter 3 Quiz
	H Getting Ready for Chapter 3 Homework
	Q Getting Ready for Chapter 4 Quiz
	H Getting Ready for Chapter 4 Homework

- **Skill Builder** offers adaptive practice that is designed to increase students' ability to complete their assignments. By monitoring student performance on homework, Skill Builder adapts to each student's needs and provides just-in-time, in-assignment practice to help enhance the student's proficiency in meeting key learning objectives.

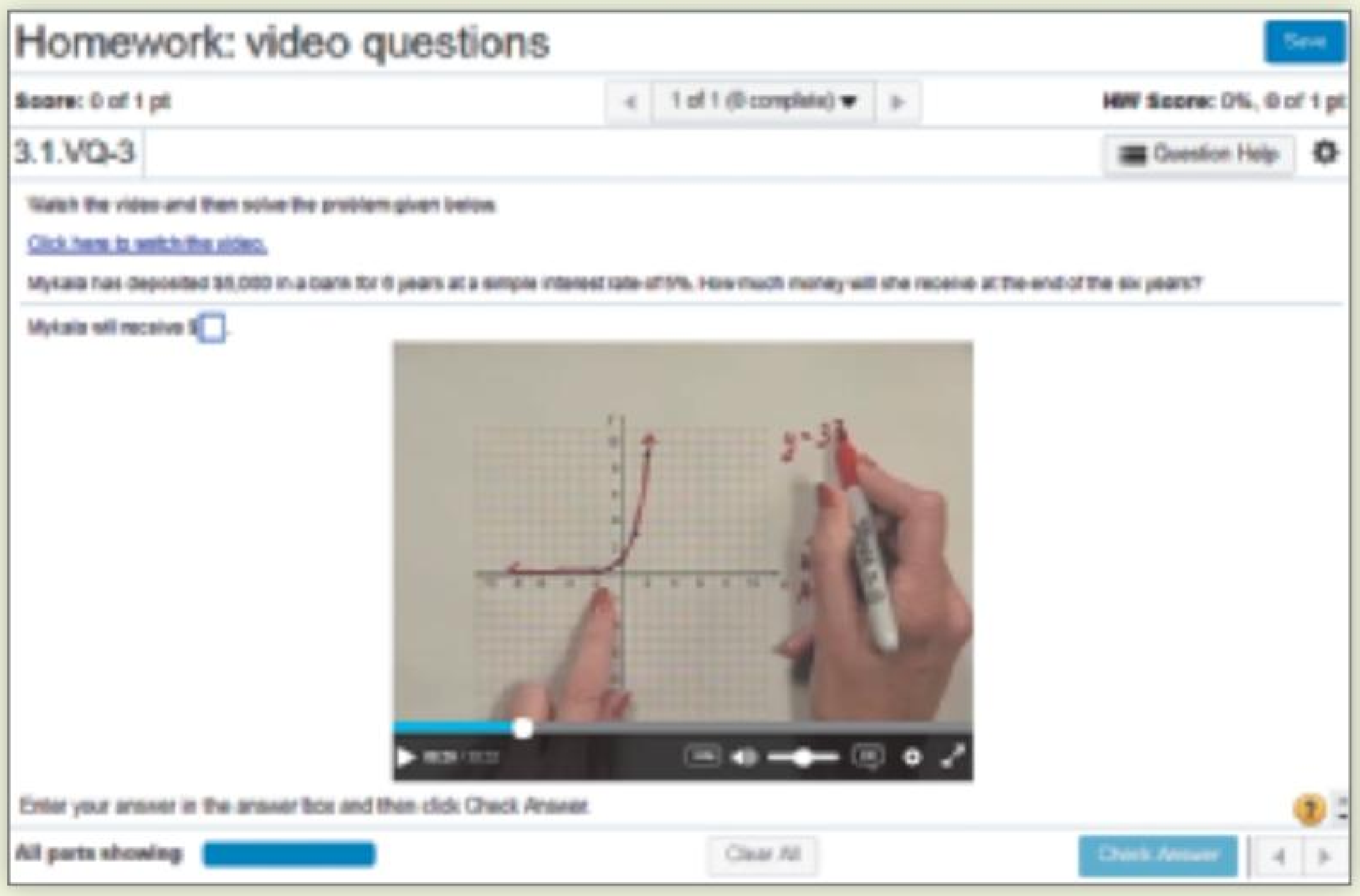
Resources for Success

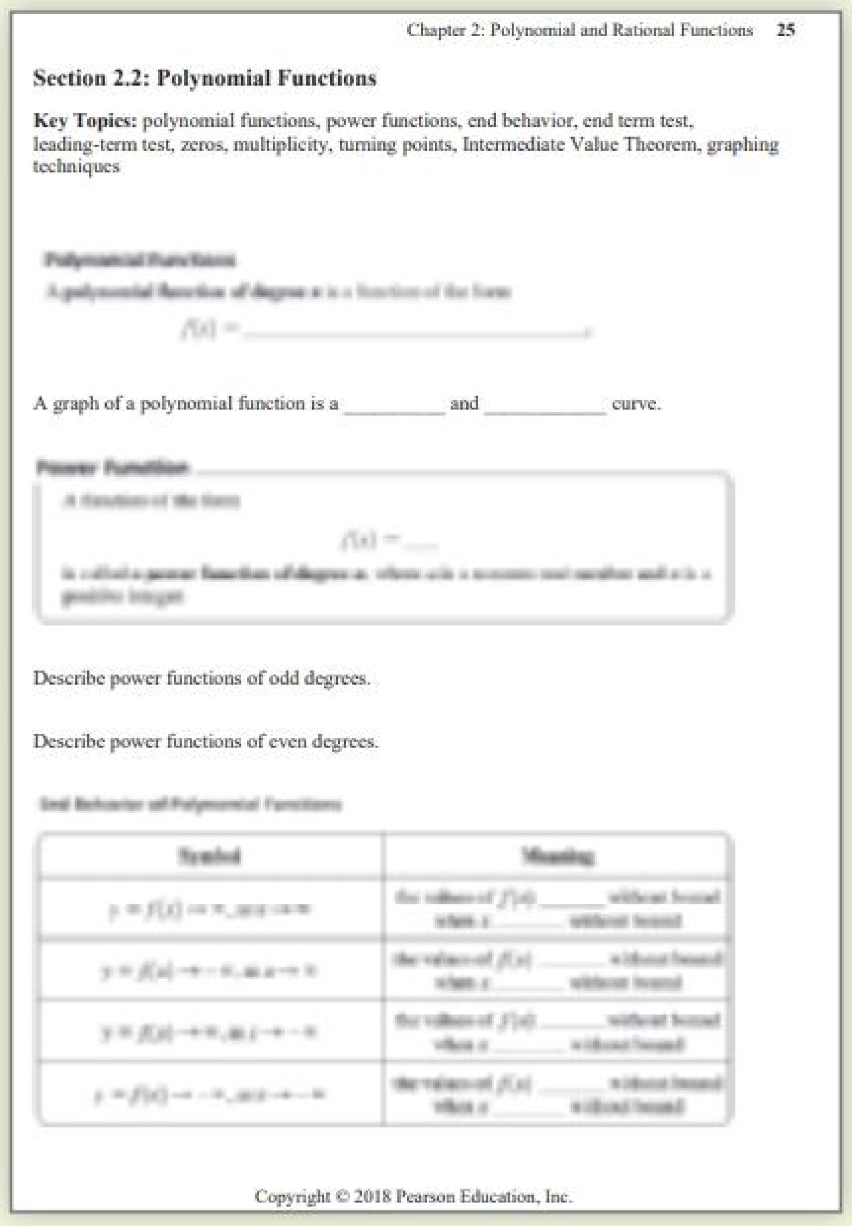
MyLab Math Online Course for *College Algebra and Trigonometry,* 4th ed., by J. S. Ratti, Marcus McWaters, and Lesław Skrzypek (access code required)

MyLab Math is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method each text's flavor and approach is tightly integrated throughout the accompanying MyLab Math course, making learning the material as seamless as possible.

Video Assessment Questions

Video Assessment questions are assignable MyLab Math exercises tied to Example Solution videos. The questions are designed to check students' understanding of the important math concepts covered in the video. The videos and Video Assessment questions provide an active learning environment where students can work at their own pace.




Chapter 2: Polynomial and Rational Functions 25

Section 2.2: Polynomial Functions

Key Topics: polynomial functions, power functions, end behavior, end term test, leading-term test, zeros, multiplicity, turning points, Intermediate Value Theorem, graphing techniques

A graph of a polynomial function is a ________ and __________ curve.

Describe power functions of odd degrees.

Describe power functions of even degrees.

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Video Notebook

The Video Notebook is a guide that gives students a structured place to take notes and work on the example problems as they watch the videos. Definitions and important concepts are highlighted, and helpful hints are pointed out along the way. The notebook is available in MyLab Math for download.

Concepts and Vocabulary Exercises

Each exercise section begins with exercises that assess the student's grasp of the definitions and ideas introduced in that section. These true-false and fill-in-the-blank exercises help to rapidly identify gaps in comprehension of the material in that section and are assignable in MyLab Math and Learning Catalytics.

pearson.com/mylab/math

Resources for Success

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Instructor Resources

Additional resources can be downloaded from **www.pearson.com** or hardcopy resources can be ordered from your sales representative.

Annotated Instructor's Edition

Answers are included on the same page beside the text exercises, where possible, for quick reference.

Instructor's Solutions Manual

Written by Beverly Fusfield, the Instructor's Solutions Manual provides complete solutions for all end-of-section exercises, including the Critical Thinking/Discussion/Writing Projects, Practice Problems, Chapter Review exercises, Practice Tests, and Cumulative Review problems.

Instructor's Testing Manual

The Instructor's Testing Manual includes diagnostic pretests, chapter tests, and additional test items, grouped by section, with answers provided.

TestGen®

TestGen® (pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

PowerPoint® Lecture Slides

The PowerPoint Lecture Slides feature presentations written and designed specifically for this text, including figures and examples from the text.

Sample Assignments in MyLab Math

Enhanced Sample Assignments created by the authors make course set-up easier by giving instructors a starting point for each chapter. Each assignment, handpicked by the author team to align with this text, includes a thoughtful mix of question types (e.g., conceptual, skills) specific to that topic. Each assignment includes new MyLab Math question types, such as vocabulary, video assessments, and set up and solve questions.

Student Resources

Additional resources are available to help students succeed.

Video Lectures

Over 25 hours of video instruction feature Section Summaries and Example Solutions. Section Summaries cover key definitions and procedures for most sections. Example Solutions walk students through the detailed solution process for many examples in the textbook. Optional subtitles are available in English and Spanish.

Student's Solutions Manual

Written by Beverly Fusfield, the Student's Solutions Manual provides detailed worked-out solutions to the odd-numbered end-of-section and Chapter Review exercises and solutions to all of the Practice Problems, Practice Tests, and Cumulative Review problems. Also available in MyLab Math.

Video Notebook

The Video Notebook is a note-taking guide that gives students a structured place to take notes and work the example problems as they watch the videos. Definitions and important concepts are highlighted, and helpful hints are pointed out along the way. The Video Notebook is available as PDFs and customizable Word files in MyLab Math.

ACKNOWLEDGMENTS

We would like to express our gratitude to the reviewers of this fourth edition (marked with an asterisk) and previous editions, who provided such invaluable insights and comments. Their contributions helped shape the development of the text and carry out the vision stated in the Preface.

Alison Ahlgren, University of Illinois at Urbana-Champaign
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Mario Barrientos, Angelo State University
Ratan Barua, Miami-Dade College
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Our sincerest thanks go to the legion of dedicated individuals who worked tirelessly to make this book possible. We would like to express our gratitude to our typist, Beverly DeVine-Hoffmeyer, for her amazing patience and skill. We must also thank Dr. Praveen Rohatgi, Dr. Nalini Rohatgi, and Dr. Bhupinder Bedi for the consulting they provided on all material relating to medicine. We particularly want to thank Professors Mile Krajcevski, Viktor Maymeskul, and Scott Rimbey for many helpful discussions and suggestions, particularly for improving the exercise sets. Further gratitude is due to Irena Andreevska, Gokarna Aryal, Ferene Tookos, and Christine Fitch for their assistance on the answers to the exercises in the text. In addition, we would like to thank De Cook, Doug Ewert, John Samons, and Beverly Fusfield, for their meticulous accuracy in checking the text. Thanks are due as well to Chere Bemelmans and Cenveo for their excellent production work. Finally, our thanks are extended to the professional and remarkable staff at Pearson Education. In particular, we would like to thank Anne Kelly, Editor in Chief; Ashley Gordon, Portfolio Management Assistant; Rachel Reeve, Content Producer; Scott Disanno, Managing Producer; Peggy Lucas, Marketing Manager; Stacey Sveum, Product Marketing Manager; Shannon McCormack, Marketing Assistant; Jonathan Wooding, Producer; Kristina Evans, Manager: Content Development, Math; Eric Gregg, Senior Content Developer, Math; and Joe Vetere, Senior Author Support Technology Specialist.

We invite all who use this book to send suggestions for improvements to Marcus McWaters at mmm@usf.edu.

Dedication

To Our Wives,

Lata, Debra, and Leslie

CHAPTER P

Basic Concepts of Algebra

TOPICS

Many fascinating patterns in human and natural processes can be described in the language of algebra. We investigate events ranging from chirping crickets to the behavior of falling objects.

SECTION P.1

The Real Numbers and Their Properties

BEFORE STARTING THIS SECTION, REVIEW FROM YOUR PREVIOUS MATHEMATICS TEXTS

1. Arithmetic of signed numbers
2. Arithmetic of fractions
3. Long division involving integers
4. Decimals
5. Arithmetic of real numbers

OBJECTIVES

1. Classify sets of real numbers.
2. Use exponents.
3. Use the ordering of the real numbers.
4. Specify sets of numbers in roster or set-builder notation.
5. Use interval notation.
6. Relate absolute value and distance on the real number line.
7. Use the order of operations in arithmetic expressions.
8. Identify and use properties of real numbers.
9. Evaluate algebraic expressions.

Cricket Chirps and Temperature

Crickets are sensitive to changes in air temperature; their chirps speed up as the temperature gets warmer and slow down as it gets cooler. It is possible to use the chirps of the male snowy tree cricket (*Oecanthus fultoni*), common throughout the United States, to gauge temperature. (The insect is found in every U.S. state except Hawaii, Alaska, Montana, and Florida.) By counting the chirps of this cricket, which lives in bushes a few feet from the ground, you can gauge temperature. Snowy tree crickets are more accurate than most cricket species; their chirps are slow enough to count, and they synchronize their singing. To convert cricket chirps to degrees Fahrenheit, count the number of chirps in 14 seconds and then add 40 to get the temperature. To convert cricket chirps to degrees Celsius, count the number of chirps in 25 seconds, divide by 3, and then add 4 to get the temperature. In Example 12, we evaluate algebraic expressions to learn the temperature from the number of cricket chirps.

1 Classify sets of real numbers.

Classifying Numbers

In algebra, we use letters such as a, b, x, y, and so on, to represent numbers. A letter that is used to represent one or more numbers is called a **variable**. A **constant** is a specific number such as 3 or $\frac{1}{2}$ or a letter that represents a fixed (but not necessarily specified) number. Physicists use the letter c as a constant to represent the speed of light ($c \approx 300{,}000{,}000$ meters per second).

We use two variables, a and b, to denote the results of the operations of addition $(a + b)$, subtraction $(a - b)$, multiplication $(a \times b$ or $a \cdot b)$, and division $\left(a \div b \text{ or } \frac{a}{b}\right)$. These operations are called **binary operations** because each is performed on two numbers.

SIDE NOTE

Here is one difficulty with attempting to divide by 0: If, for example, $\frac{5}{0} = a$, then $5 = a \cdot 0$. However, $a \cdot 0 = 0$ for all numbers a. So we would have $5 = 0$; this contradiction demonstrates that there is no appropriate choice for $\frac{5}{0}$.

We frequently omit the multiplication sign when writing a product involving two variables (or a constant and a variable) so that $a \cdot b$ and ab indicate the same product. Both a and b are called **factors** in the product $a \cdot b$. This is a good time to recall that we never divide by zero. For $\frac{a}{b}$ to represent a real number, b cannot be zero.

Equality of Numbers

The **equal sign**, $=$, is used much like we use the word *is* in English. The equal sign means that the number or expression on the left side is equal or equivalent to the number or expression on the right side. We write $a \neq b$ to indicate that a is not equal to b.

Classifying Sets of Numbers

The idea of a set is familiar to us. We regularly refer to "a set of baseball cards," a "set of CDs," or "a set of dishes." In mathematics, as in everyday life, a **set** is a collection of objects. The objects in the set are called the **elements**, or **members**, of the set. Capital letters are usually used to name a set. In the study of algebra, we are interested primarily in sets of numbers.

In listing the elements of a set, it is customary to enclose the listed elements in braces, $\{\ \}$, and separate them with commas.

We distinguish among various sets of numbers.

The numbers we use to count with constitute the set N of **natural numbers**: $\text{N} = \{1, 2, 3, 4, \ldots\}$.

The three dots ... (called ellipsis) may be read as "and so on" and indicate that the pattern continues indefinitely.

The set W of **whole numbers** is formed by including the number 0 with the natural numbers to obtain the set: $\text{W} = \{0, 1, 2, 3, 4, \ldots\}$.

The set Z of **integers** consists of the set N of natural numbers together with their opposites and 0: $\text{Z} = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$.

Rational Numbers

The **rational numbers** consist of all numbers that *can* be expressed as the quotient or ratio, $\frac{a}{b}$, of two integers, where $b \neq 0$. The letter Q is often used to represent the set of rational numbers.

Examples of rational numbers are $\frac{1}{2}, \frac{5}{3}, \frac{-4}{17}$, and $0.07 = \frac{7}{100}$. Any integer a can be expressed as the quotient of two integers by writing $a = \frac{a}{1}$. Consequently, every integer is also a rational number. In particular, 0 is a rational number because $0 = \frac{0}{1}$.

The rational number $\frac{a}{b}$ can be written as a decimal by using long division. When any integer a is divided by an integer b, $b \neq 0$, the result is always a **terminating decimal** such as $\left(\frac{1}{2} = 0.5\right)$ or a **nonterminating repeating decimal** such as $\left(\frac{2}{3} = 0.666\ldots\right)$.

We sometimes place a bar over the repeating digits in a nonterminating repeating decimal. Thus, $\frac{2}{3} = 0.666\ldots = 0.\overline{6}$ and $\frac{141}{110} = 1.2818181\ldots = 1.2\overline{81}$.

EXAMPLE 1 **Converting Decimal Rationals to a Quotient**

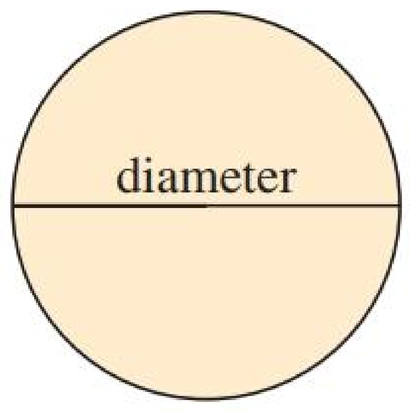
Write the rational number $7.\overline{45}$ as the ratio of two integers in lowest terms.

Solution

Let $x = 7.454545\ldots$. Then

$$\begin{array}{rrll} & 100x = & 745.4545\ldots & \text{Multiply both sides by 100.} \\ \text{subtract} & x = & 7.4545\ldots & \\ \hline & 99x = & 738 & 100x - x = 99x \\ & x = & \dfrac{738}{99} & \text{Divide both sides by 99.} \\ & = & \dfrac{82 \times \cancel{9}}{11 \times \cancel{9}} & \text{Common factor} \\ & x = & \dfrac{82}{11} & \text{Reduce to lowest terms.} \end{array}$$

Practice Problem 1 Repeat Example 1 for $2.132132132\ldots$.



$\pi = \dfrac{\text{circumference}}{\text{diameter}}$

Figure P.1 Definition of π

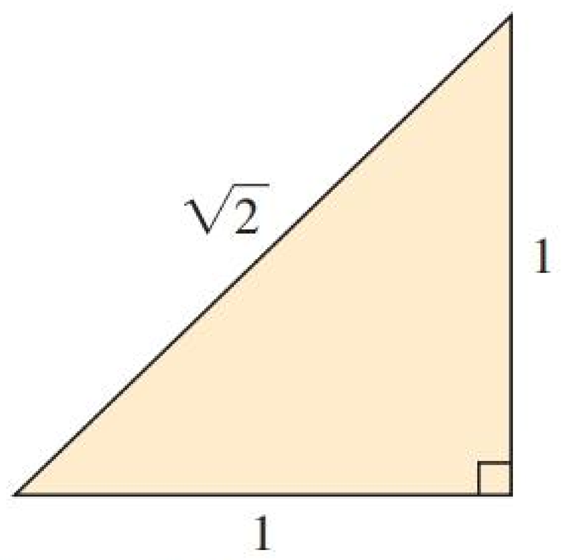


Figure P.2

Irrational Numbers

An **irrational number** is a number that cannot be written as a ratio of two integers. This means that its decimal representation must be nonrepeating and nonterminating. We can construct such a decimal using only the digits 0 and 1, such as $0.01001000100001\ldots$. Because each group of zeros contains one more zero than the previous group, no group of digits repeats. Other numbers such as π ("pi"; see Figure P.1) and $\sqrt{2}$ (the square root of 2; see Figure P.2) can also be expressed as decimals that neither terminate nor repeat; so they are irrational numbers as well. We can obtain an approximation of an irrational number by using an initial portion of its decimal representation. For example, we can write $\pi \approx 3.14159$ or $\sqrt{2} \approx 1.41421$, where the symbol $\approx$ is read "is approximately equal to."

No familiar process, such as long division, is available for obtaining the decimal representation of an irrational number. However, your calculator can provide a useful approximation for irrational numbers such as $\sqrt{2}$. (Try it!) Because a calculator displays a fixed number of decimal places, it gives a **rational approximation** of an irrational number.

It is usually not easy to determine whether a specific number is irrational. One helpful fact in this regard is that *the square root of any natural number that is not a perfect square is irrational.* So $\sqrt{6}$ is irrational but $\sqrt{16} = \sqrt{4^2} = 4$ is rational.

RECALL

An integer is a *perfect square* if it is a product $a \cdot a$, where a is an integer. For example, $9 = 3 \cdot 3$ is a perfect square.

Because rational numbers have decimal representations that either terminate or repeat, whereas irrational numbers do not have such representations, *no number is both rational and irrational.*

The rational numbers together with the irrational numbers form the set R of **real numbers**.

The diagram in Figure P.3 shows how various sets of numbers are related. For example, every natural number is also a whole number, an integer, a rational number, and a real number.

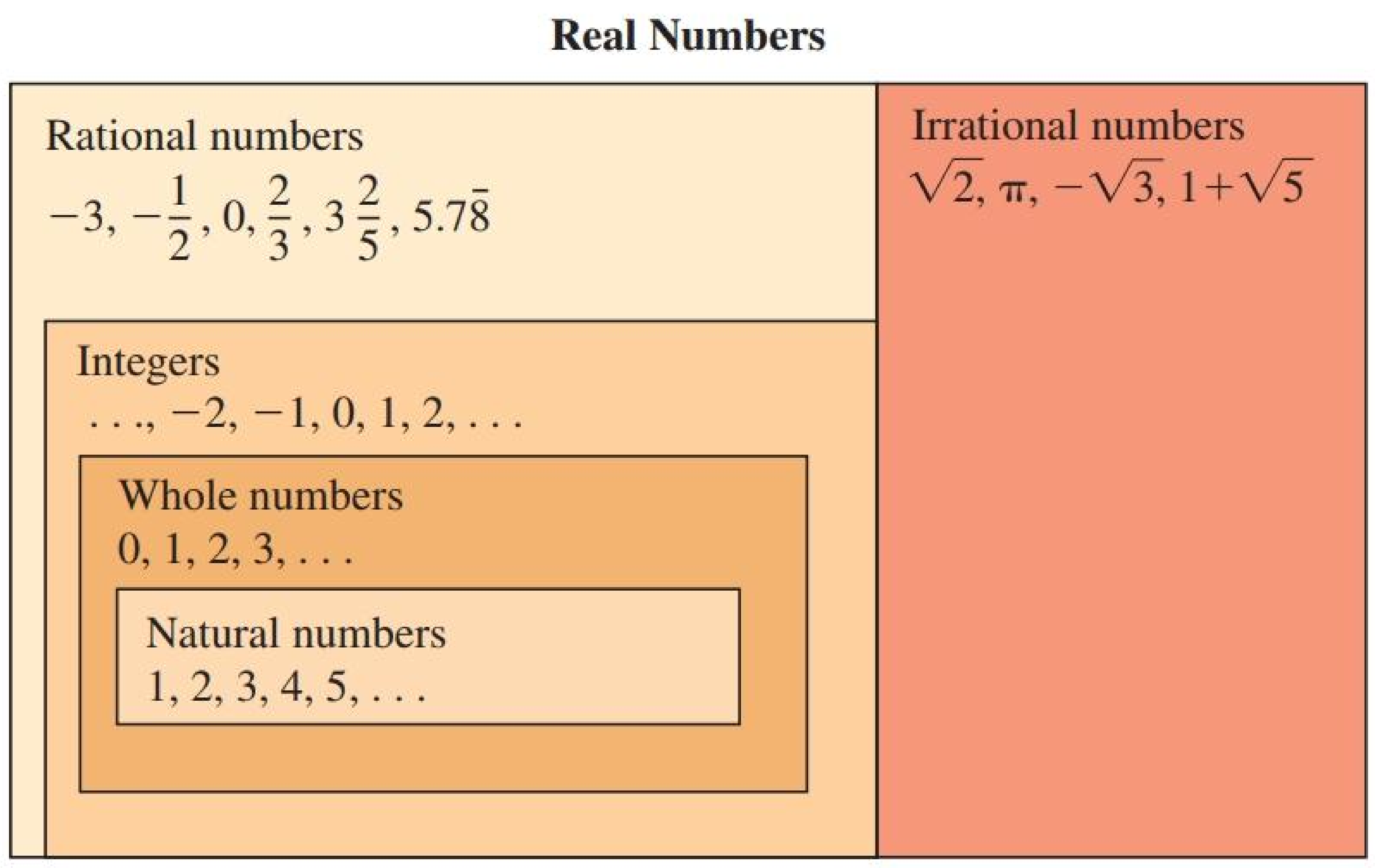


Figure P.3 Relationships among sets of real numbers

EXAMPLE 2 **Identifying Sets of Numbers**

Let A $= \left\{-17, -5, -\frac{6}{3}, -\frac{2}{3}, 0, \frac{5}{12}, \frac{1}{2}, \sqrt{2}, \pi, \sqrt{35}, 7, 18\right\}$.

Identify all the elements of the set A that are:

a. Natural numbers **b.** Whole numbers **c.** Integers
d. Rational numbers **e.** Irrational numbers **f.** Real numbers

Solution

a. Natural numbers: 7 and 18

b. Whole numbers: 0, 7, and 18

c. Integers: $-17, -5, -\frac{6}{3}$ (or -2), 0, 7, and 18

d. Rational numbers: $-17, -5, -\frac{6}{3}, -\frac{2}{3}, 0, \frac{5}{12}, \frac{1}{2}$, 7, and 18

e. Irrational numbers: $\sqrt{2}$, π, and $\sqrt{35}$

f. Real numbers: All numbers in the set A are real numbers.

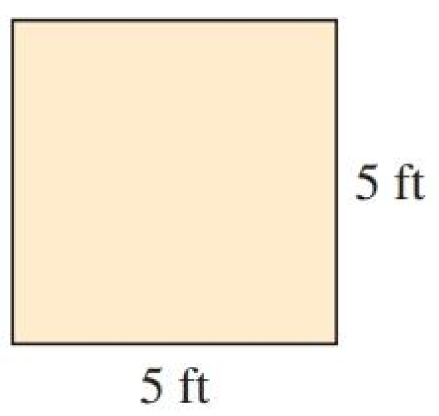
Practice Problem 2 Repeat Example 2 for the set:

$$B = \left\{-6, -\frac{21}{7}, -\frac{1}{2}, 0, \frac{4}{3}, \sqrt{3}, 2, \sqrt{17}, 7\right\}.$$

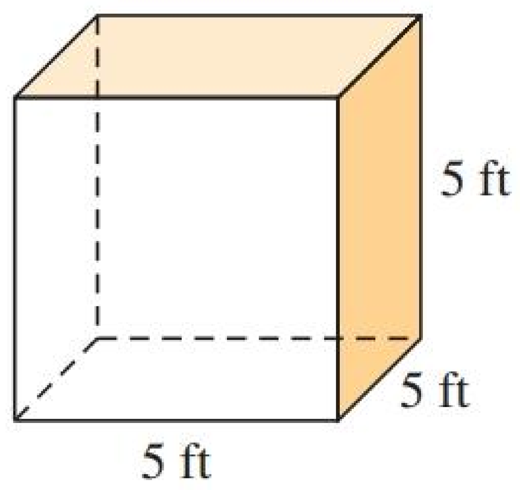
2 Use exponents.

Integer Exponents

The area of a square with side 5 feet is $5 \cdot 5 = 25$ square feet.
The volume of a cube that has sides of 5 feet each is $5 \cdot 5 \cdot 5 = 125$ cubic feet.



Area = 5·5 square feet



Volume = 5·5·5 cubic feet

A shorter notation for $5 \cdot 5$ is 5^2 and for $5 \cdot 5 \cdot 5$ is 5^3. The number 5 is called the *base* for both 5^2 and 5^3. The number 2 is called the *exponent* in the expression 5^2 and indicates that the base 5 appears as a factor twice.

Positive Integer Exponent

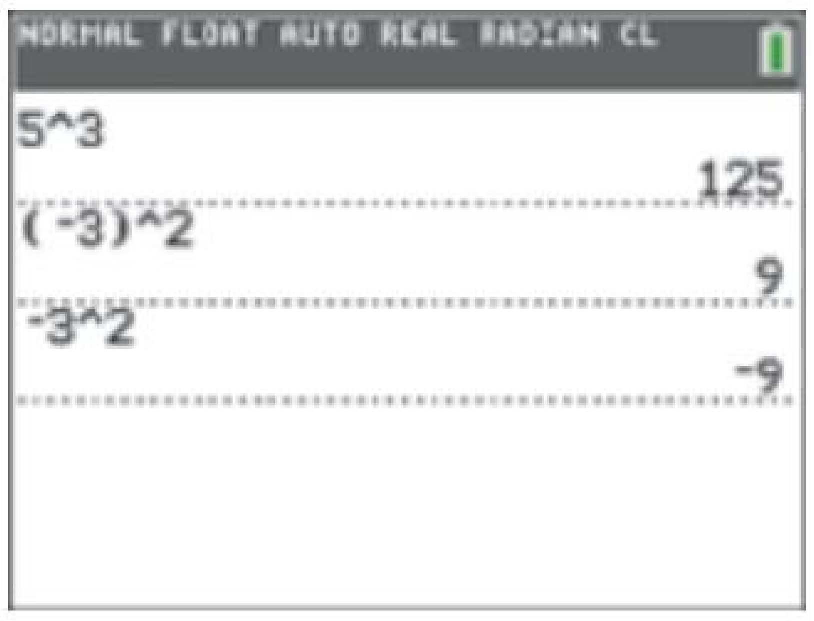
If a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}.$$

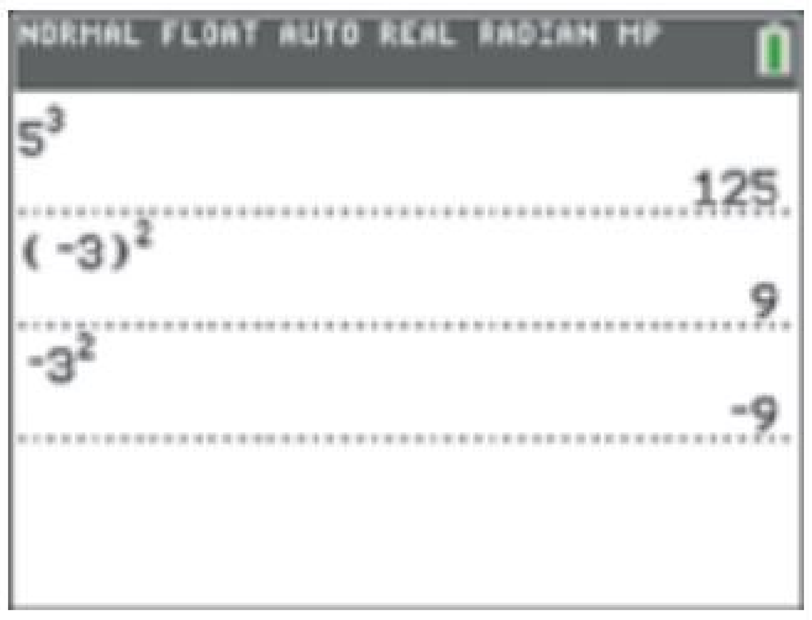
The number a^n is called the nth power of a and is read "a to the nth power", or "a to the n". The number a is called the **base**; n, the **exponent**. We adopt the convention that $a^1 = a$.

TECHNOLOGY CONNECTION

Type 5^3 for 5^3 on a graphing calculator. Any expression on a calculator enclosed in parentheses and followed by ^*n* is raised to the *n*th power. A common error is to forget parentheses when computing an expression such as $(-3)^2$, with the result being -3^2. Newer graphing calculators, such as the TI-84 series, provide display options called $\boxed{\text{CLASSIC}}$ and $\boxed{\text{MATHPRINT}}$. The MATHPRINT option displays 5^3 as 5^3 but still requires you to type 5, then ^, then 3.



$\boxed{\text{CLASSIC}}$ Mode



$\boxed{\text{MATHPRINT}}$ Mode

In this text we will display screens in MATHPRINT mode.

EXAMPLE 3 Evaluating Expressions That Use Exponents

Evaluate each expression.

a. 5^3 **b.** $(-3)^2$ **c.** -3^2 **d.** $(-2)^3$

Solution

a. $5^3 = 5 \cdot 5 \cdot 5 = 125$

b. $(-3)^2 = (-3)(-3) = 9$ $\quad (-3)^2$ is the opposite of 3, squared.

c. $-3^2 = -3 \cdot 3 = -9$ $\quad$ Multiplication of $3 \cdot 3$ occurs first. (-3^2 is the opposite of 3^2.)

d. $(-2)^3 = (-2)(-2)(-2) = -8$

Practice Problem 3 Evaluate the following.

a. 2^3 **b.** $(3a)^2$ **c.** $\left(\frac{1}{2}\right)^4$

In Example 3, pay careful attention to the fact (from parts **b** and **c**) that $(-3)^2 \neq -3^2$. In $(-3)^2$, the parentheses indicate that the exponent 2 applies to the base -3, whereas in -3^2, the absence of parentheses indicates that the exponent applies only to the base 3. When n is even, $(-a)^n \neq -a^n$ for $a \neq 0$.

The Real Number Line

We associate the real numbers with points on a geometric line (imagined to be extended indefinitely in both directions) in such a way that each real number corresponds to exactly one point and each point corresponds to exactly one real number. The point is called the **graph** of the corresponding real number, and the real number is called the **coordinate** of the point. By agreement, ***positive numbers*** lie to the right of the point corresponding to 0 and ***negative numbers*** lie to the left of 0. See Figure P.4.

Notice that $\frac{1}{2}$ and $-\frac{1}{2}$, 2 and -2, and π and $-\pi$ correspond to pairs of points exactly the same distance from 0 but on opposite sides of 0.

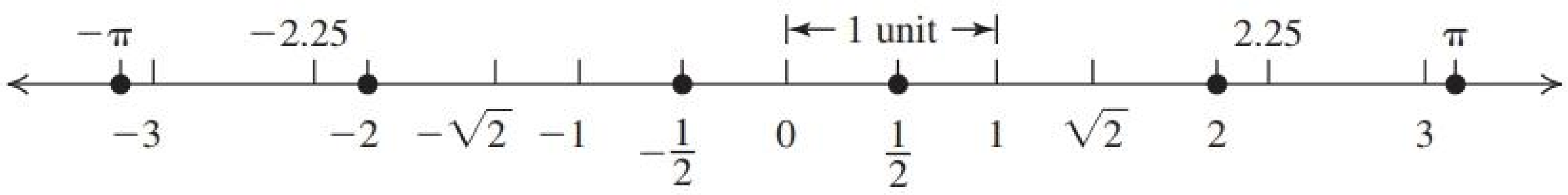


Figure P.4 The real number line

When coordinates have been assigned to points on a line in the manner just described, the line is called a **real number line**, a **coordinate line**, a **real line**, or simply a **number line**. The point corresponding to 0 is called the **origin**.

3 Use the ordering of the real numbers.

Inequalities

The real numbers are **ordered** by their size. We say that ***a* is less than *b*** and write $a < b$, provided that $b = a + c$ for some ***positive*** number c. We also write $b > a$, meaning the same thing as $a < b$, and say that ***b* is greater than *a***. On the real line, the numbers increase from left to right. Consequently, ***a is to the left of b on the number line when a < b***. Similarly, a is to the right of b on the number line when $a > b$. We sometimes want to indicate that at least one of two conditions is correct: Either $a < b$ or $a = b$. In this case, we write $a \leq b$ or $b \geq a$. The four symbols $<, >, \leq$, and $\geq$ are called **inequality symbols**.